European Credit Views

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Trading correlation in the euro area CDO

End of the world looks almost fully priced in!

Relative to Italy and Spain, the AAA tranches look too tight

On 21 May 2010, we introduced the idea that EMU looks in some respects like a CDO[1]. We have since extended the idea to a recommendation to "sell the AAA tranche", i.e. buy sovereign CDS protection on Germany and other AAA rated sovereigns[2]. These trades have performed well; Germany CDS is trading close to its all time wides and several other "low beta" sovereigns are also close to their peaks.

We think that the sovereign debt crisis is nowhere near its end as we are dealing with an array of 17 governments and several sub bodies that – for game theoretical reasons[3] – will struggle to speak credibly with one voice before the market pushes them to the very edge.

Outright, it might therefore very well make sense to hold onto those shorts in the "senior and super senior tranches". After the recent rally, entry points are even emerging again. Furthermore, on a relative value basis, we think that these two look relatively expensive (from a risk perspective) in comparison to the "senior mezzanine tranche", namely Italy and Spain.

In our opinion there are three different ways how to trade these tranches:

1. Buy and hold. A buy and hold investor should make sure they can withstand potentially large mark-to-market volatility in spreads and should confine long risk positions to sovereigns that are not going to default. In our opinion these are all governments in the euro, except for Greece and possibly Portugal.
2. Directional mark-to-market play: In this set-up we feel that the current huge volatility in all sovereign CDS warrants tactical shorts. We continue to think the senior tranches are the cheapest shorts by a measure of spread level vs. spread widening potential.
3. Relative value view on correlation: We think that the expected loss of the "senior mezzanine tranche" should decrease again. We therefore recommend going long risk in Spain and Italy vs. short risk in Germany or other AAA sovereign CDS, delta-adjusted.

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1 European Credit Flash: Life is a CDO old chum, 21 May 2010

2 European Credit Flash: Sell the AAA "tranche", 15 July 2011;
European Credit Cash Weekly: Underperforming senior "tranche", 01 August 2011;
Credit Index Analyser: Low beta sovereigns to widen further, 02 August 2011

3 A Nash Equilibrium for Greece: Applying Game Theory to the Greek debt crisis, 15 June 2011

Introduction

On 21 May 2010, we introduced the idea that EMU looks in some respects like a CDO[4]. We have since extended the idea to a recommendation to "sell the AAA tranche", i.e. buy sovereign CDS protection on Germany and other AAA rated sovereigns[5].

We have found this analytical framework useful and now try to attach some rigour to it. We explain the mathematical concepts and tools to evaluate tranches thereby taking into account the crucially important concept of tail dependence. We use the model to derive implied correlations of the euro area CDO, as priced by the single name sovereign CDS market. The history of these correlations and their resultant Expected Loss (EL) series provide the necessary tool to provide relative value recommendations in the sovereign CDS market, as well as a useful concept to understand what is going on; we receive many questions about German asset swaps and CDS, for example.

The strategic background: euro/EMU

Our recently-published "World in pictures" entitled "European credit during the bank/sovereign loop[6]" goes in some detail into this area. Very broadly, we see a previously unacknowledged cost offsetting the net benefit of the euro. The "austerity vs. Eurobonds" debate, to stylize extremely, is about where this cost is recognised. With the periphery at its limits, we think some cost inevitably comes to the core; the question is how. Simply through the banking system, the public sector (fiscal transfers) or a bit of both (the Eurosystem). If costs are imposed through fiscal transfers (at the extreme, immediate joint and several guarantees) then correlations rise; if they are imposed through imposing default costs on the private sector, then they fall. Our CDO framework provides a way of substantiating the considerable verbiage surrounding these issues.

Framework of the CDO

The key ingredients to a CDO are the so called attachment and detachment points, which determine the degree of subordination a tranche holds as a buffer against losses in the underlying pool of credits. When comparing the euro area to a CDO, one therefore needs to define what the respective tranches represent. As we have described in our *European Credit Flash: Sell the AAA "tranche",* we use the largest European countries on which sovereign CDS are traded[7] and sort them by their 5y CDS spread. We overrule this order only in the case of Germany, which given the size of its economy and role as issuer of "risk free" bonds in the sovereign bond market, we see as the super senior tranche. We then group countries that the market trades in close partnership into the same tranche with the following representations:

- Greece is the equity tranche and for many reasons we think it is the sole country in this tranche
- Portugal and Ireland trade in a very similar vein although the reasons for their very wide spreads are different (for instance Ireland has to deal more with an inherited private sector debt problem, whereas Portugal with a public sector debt issue). We therefore place them into the junior mezzanine tranche
- Italy and Spain are also very closely related. Italy used to trade inside Spain, however recently this relationship has inverted. We think that this is the key tranche that may never be "touched" in order for the euro project to continue

4 European Credit Flash: Life is a CDO old chum, 21 May 2010

5 European Credit Flash: Sell the AAA "tranche", 15 July 2011;
European Credit Cash Weekly: Underperforming senior "tranche", 01 August 2011;
Credit Index Analyser: Low beta sovereigns to widen further, 02 August 2011

6 European Credit during the bank/sovereign loop

7 As far as we are aware there is no liquid market in Luxembourg CDS

successfully. In our view, the ECB more or less has to **define** financial stability as a situation where Italy and Spain are par assets and then work from there. Belgium trades somewhat tighter, but based on its spread level and rating does not qualify for the senior tranche. We therefore aggregate these three countries into the senior mezzanine tranche.

- The senior tranche consists of all AAA rated countries, except for Germany, i.e. The Netherlands, France, Finland and Austria
- As explained above, Germany represents the super senior tranche[8]

Furthermore, we use the ECB capital shares of each of these countries to define attachment and detachment points.

Exhibit 1: The tranches of the Euro CDO

S&P	Moody's	NCB	Capital key %	Paid-up capital (€)	Share	Attach	Detach
AAA	Aaa	Deutsche Bundesbank	18.9373	1,406,533,694.10	28%	72.3%	100.0%
AAA	Aaa	Suomen Pankki - Finlands Bank	1.2539	93,131,153.81	2%	70.5%	72.3%
AAA	Aaa	De Nederlandsche Bank	3.9882	296,216,339.12	6%	64.6%	70.5%
AAA	Aaa	Oesterreichische Nationalbank	1.9417	144,216,254.37	3%	61.8%	64.6%
AAA	Aaa	Banque de France	14.2212	1,056,253,899.48	21%	41.0%	61.8%
AA+	Aa1	Nationale Bank van België / Banque Nationale de Belgique	2.4256	180,157,051.35	4%	37.5%	41.0%
A+	Aa2	Banca d'Italia	12.4966	928,162,354.81	18%	19.2%	37.5%
AA	Aa2	Banco de España	8.304	616,764,575.51	12%	7.1%	19.2%
BBB+	Ba1	Central Bank of Ireland	1.1107	82,495,232.91	2%	5.4%	7.1%
BBB-	Ba2	Banco de Portugal	1.7504	130,007,792.98	3%	2.9%	5.4%
CC	Ca	Bank of Greece	1.9649	145,939,392.39	3%	0.0%	2.9%

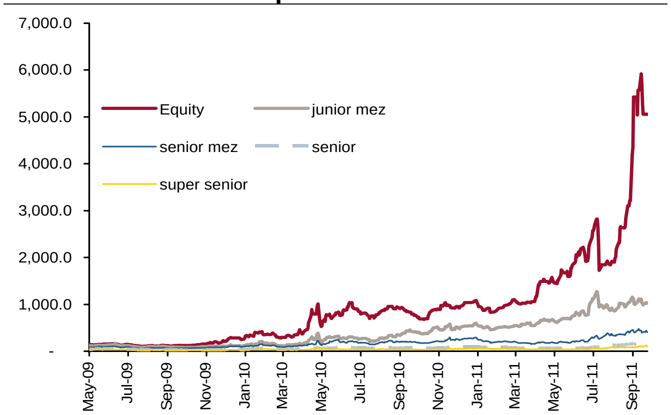
Source: Credit Suisse

Tranche spread performance

Exhibit 2 illustrates the historical spread performance of the five tranches since May 2009. While Greece has consistently made new wides, our recommendation to sell the AAA tranche also has performed very well since inception on 15 July 2011 as is illustrated in Exhibit 3.

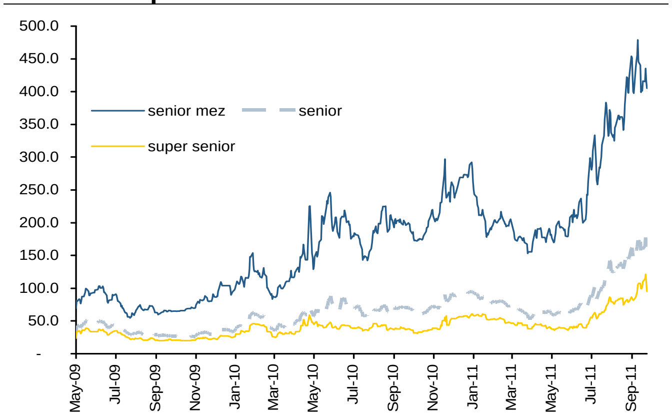
We think this trend is likely to continue until we enter a new stage of the crisis, e.g. a Greek default. At that stage it will be crucial to see if further contagion will be allowed to spread to the junior mezzanine tranche or, via the banking sector, to more senior tranches and how officials then react to that. Ultimately, we believe the ECB will have to step up and provide a credible and unconditional backstop to Spanish and Italian spreads and therefore prevent the crisis escalating to the senior mezzanine tranche.

Exhibit 2: Historical spreads of the five tranches



Source: Credit Suisse

Exhibit 3: Spreads of the three most senior tranches



Source: Credit Suisse

8 We also have run a separate model in which Germany is part of the senior tranche leading to very similar results. The additional super senior tranche however provides for an interesting additional layer in our opinion.

But this publication is not so much about directional trades but more about the relative value between these tranches. In order to understand where these lie, we need to extract the implied correlations the market is pricing according to the spreads these tranches trade at relative to each other. In the following chapters, we provide the basic modeling framework to derive these, with a detailed mathematical explanation in the appendix.

Implied correlations

The concept of implied correlation is based on the fact that the model price of any tranche is a function of its attachment and detachment points, the default probability and recovery rates of the pool of underlyings and, probably most importantly, the dependency structure between defaults of individual underlyings[9]. This default dependency is commonly referred to as default correlation[10] since in the most commonly used model, the Gaussian copula, single-factor model, the default dependency between two issuers is determined by the linear correlation coefficient between their asset values. Given all the other input parameters of the model, including the tranche's market price it therefore is possible to derive the implied correlation between the issuers in the pool[11]. In other words, once you know the attachment point A, detachment point D, default probabilities P[12] and recovery rates R of each underlying and the Market price of the tranche, you can vary the correlation coefficient Rho in order to make the below equation hold:

$$Modelprice(A, D, P, R, Rho) = Marketprice$$

In the following sections we will start with the standard, Large Homogeneous Portfolio (LHP) model with a Gaussian Copula and a standard fixed recovery rate of 40%. We will also show how that model is incapable of producing model spreads that are as wide as market spreads for senior tranches. We will therefore, first add a so called stochastic recovery to the existing model which will alleviate this problem, but not solve it. As we will see, the most senior tranches currently price in a substantial amount of tail-dependency[13], which we will only be able to match by introducing a new copula, in this case the Student-t copula.

Compound correlation

Compound correlation is the implied correlation, derived separately for each tranche as a function of both the attachment and detachment points. There are two main problems with compound correlation

1. It does not preserve the Expected Loss (EL), as the sum of the EL of each tranche do not add up to the EL of the portfolio. This is a breach of a no-arbitrage constraint.
2. Less importantly, tranches with other attachment and detachment points can't be priced using the implied correlation curve.[14]

9 Other parameters also play a role, for example the risk free rate term structure and the default probability term structure. We take these into account in our model, which we explain in the appendix, however, for the sake of simplicity disregard here for the moment.

10 Technically that is wrong, as only in the Gaussian copula framework, the dependency structure is described by one single correlation parameter, namely the linear correlation coefficient. The paper "Correlation and dependency in risk management: Properties and pitfalls" by Paul Embrechts, Alexander McNeil and Daniel Straumann (2002) gives a good summary on this.

11 Assuming they all correlate equally highly with each other; this is part of the so called homogeneous default pool assumption

12 P and R are vectors, whose components are the individual default probabilities and recovery rates of each individual underlying credit.

13 Tail dependency is a copula function and describes the probability of an extreme event in one asset, conditional on the probability of an extreme move of another asset. For a proper definition see the appendix

14 For more detail on this see: Modelling single-name and multi-name Credit Derivatives by Dominic O'Kane

Base correlation

For the reasons mentioned above, the iTraxx Main and CDX IG tranche market have adopted the concept of base correlations as the standard model. Base correlation does fulfill the two requirements above and hence all correlations quoted hereafter are of the base correlation type. (iTraxx is a trademark of International Index Company Limited.)

Gaussian Copula model doesn't work

As mentioned above, the standard model with a fixed 40% recovery rate can not be calibrated to the market prices of the two most senior tranches, i.e. there is no correlation that can make the model spread match the market spread of these tranches. In the case of the super senior tranche this doesn't come as a surprise as with a 40% recovery rate, even if all underlyings defaulted immediately, only 60% of the full index notional could be lost. The super senior tranche with a attachment point of 72.3% would therefore always stay untouched.

We attempt to solve this by making the recovery rate a random variable, that is, lower when the default frequency is high (impacting the senior tranches) and higher when there are fewer defaults (impacting the junior tranches)[15]. When allowing a zero percent recovery for the highest frequency of defaults we can make the senior tranche calibrate at all times, however the super senior, i.e. Germany is still too wide to be modeled this way. As expected—we have complained about the Gaussian fallacy since 2004[16]—the reason is the Gaussian copula, which has a zero tail dependency, or in other words, come extreme events, defaults are independent. Clearly, this needs to be changed and the way we resolve this is by using a Student–t copula for modeling the dependencies between defaults in the pool. Exhibit 4 illustrates these implied base correlations under the Student-t copula model with a stochastic recovery rate floored at 0%[17].

Results and interpretation

The first observation we make is that during 2009, implied correlations across all tranches were very high and did hardly differ. I.e. the market was not really distinguishing between tranches and therefore sovereigns, as all of them were systemically important and their liabilities money good. The "Par Pretence" was more alive than ever.

The next phase started in early 2010 and back then was generally labeled a "Greek Crisis". It was characterized by a divergence in implied correlations between the different tranches and a continued fall in implied correlations for the equity tranche. In other words, the market was considering and pricing Greece risk to be of idiosyncratic nature.

This changed in September 2010, when the implied correlation of the junior mezzanine tranche (i.e. Portugal and Ireland) rapidly caught up with the equity tranche and the gap between these two collapsed in a matter of weeks. According to the market, the crisis had become more systemic, by spreading contagion from Greece to these two other peripheral sovereigns. Interestingly, the implied correlations of these two tranches would then continue to move in very close partnership until the very recent stage of the crisis.

In this (so far) latest episode, we observe that since the beginning of August 2011, firstly the implied correlation of the equity tranche virtually collapsed, pricing for an idiosyncratic risk event in Greece. Secondly, the implied correlation of the junior mez tranche followed only marginally, thereby ringfencing Ireland and Portugal from the Greek default scenario at least for the moment. And thirdly, the implied correlations of the senior mez and senior tranches have risen significantly, thereby expressing the view that we have arrived at a stage of the crisis that is purely systemic[18].

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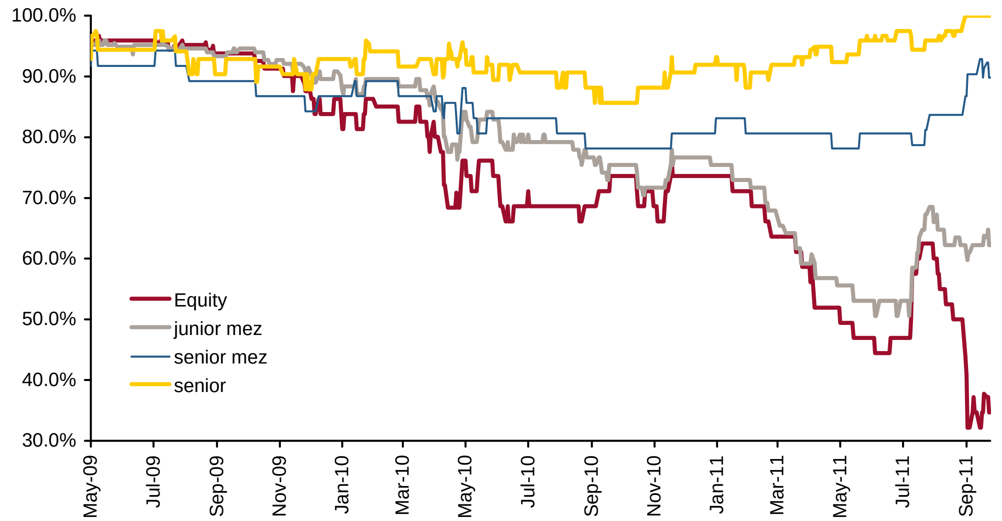
15 We follow the paper Optimal Stochastic Recovery for base Correlation by Salah Amraoui and Sebastien Hitier (2008)

16 European Credit Strategy and Trades: Tsunami

17 The corresponding charts under the Gaussian Copula can be found in the appendix.

18 Our Fear Factor analysis, which compares the spread and mark-to-market risk levels of various iTraxx indices has been showing for a long time that the ratios of iTraxx Main, Xover and Senior Financial are pricing for a very large degree of systemic risk.

Exhibit 4: Time series of implied base correlations (t-copula)



Source: Credit Suisse

Expected Losses

While we think that the above historical evolution of implied correlations is very useful in separating the crisis into its various stages, we would like to draw conclusions from potential future implied correlation moves in order to be able to produce relative value trade ideas. We therefore introduce the concept of the expected loss of a tranche.

Basically, the loss of a tranche is unknown and driven by the underlying portfolio loss. Any non-equity tranche has a chance of not incurring losses as long as the portfolio losses remain lower than the attachment point of the respective tranche. In addition, a tranche can be completely wiped out, by means of portfolio losses that are higher than its detachment point. Finally, any loss between 0% and 100% of the tranche notional has the probability of the portfolio loss being between the attachment and detachment points of the tranche. Furthermore, the sum of the expected losses of all tranches add up to the expected loss of the underlying portfolio. Crucially, however, the distribution of the portfolio expected loss across the various tranches is a function of the probability distribution of the portfolio loss and therefore the default correlation between issuers[19].

Exhibit 5 illustrates how this expected loss distribution across tranches has evolved historically. Firstly, it is important to remember that the lower two tranches are very thin (3%-4% of portfolio notional) and that the upper three tranches are roughly all equal in size (28%-34%). Secondly, the more junior any of these three senior tranches are, the more likely they are to be hit by a loss and maybe to even be wiped out. It therefore makes a lot of sense that initially, the senior mez tranche holds the largest share of the portfolio expected loss. Interestingly, however, this tranche’s expected loss gets “squeezed” from both sides – the more junior and the more senior tranches – over time. Particularly the two most senior tranches have “eaten away” large chunks of the expected loss from the senior mezzanine tranche. However, we note the very recent increase in size of the EL of the senior mezzanine tranche, which currently stands at 35.5% after having been as low as 32.5% recently.

Since in our opinion, this crisis is all about the ability of Spain to be a borrower and Italy to be a lender (Belgium probably sits in between these two categories) we think this recent EL increase of the senior mezzanine tranche should reverse again. In other words, the

[19] The mathematical definition of the expected loss of a tranche is given in the appendix.

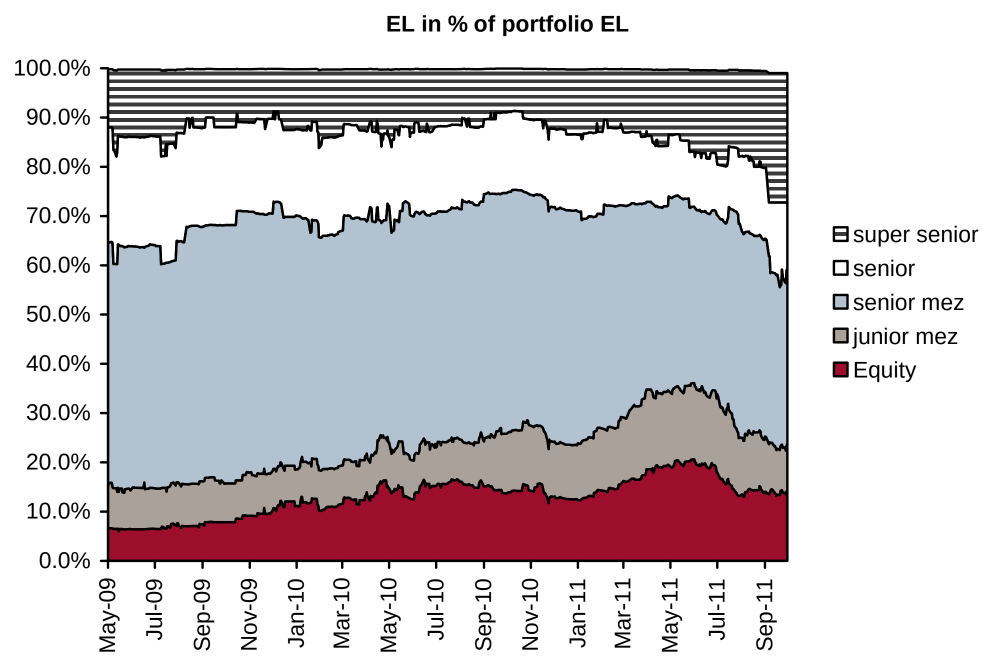
spread of the senior mezzanine tranche is too wide versus the other tranches, particularly the two more senior ones and we think it makes sense at these levels to sell protection on the basket of Italy and Spain (and maybe Belgium) and hedge it via a short risk position in CDS on Germany or other AAA sovereigns.

We continue to think that at this stage of the crisis, the senior tranches are the "cheapest hedges" by a measure of spread level vs. spread widening potential. Outright, it might therefore very well make sense to hold onto those shorts in the "senior and super senior tranches". After the recent rally, entry points are even emerging again.

In our opinion there are three different ways how to trade these tranches:

1. Buy and hold. A buy and hold investor should make sure they can withstand potentially large mark-to-market volatility in spreads and should confine long risk positions to sovereigns that are not going to default. In our opinion these are all governments in the euro, except for Greece and possibly Portugal.
2. Directional mark-to-market play: In this set-up we feel that the current huge volatility in all sovereign CDS warrants tactical shorts. We continue to think the senior tranches are the cheapest shorts by a measure of spread level vs. spread widening potential.
3. Relative value view on correlation: We think that the expected loss of the "senior mezzanine tranche" should decrease again. We therefore recommend going long risk in Spain and Italy vs. short risk in Germany or other AAA sovereign CDS.

Exhibit 5: Expected Loss distribution across tranches



Source: Credit Suisse

Hedge ratios

The last point in the previous section brings up the important issue of hedging ratios for relative value trades. In this note we provide two ways how to calculate hedging ratios

1. Model implied hedging ratios are derived from the specifications of a tranche: Attachment and Detachment points, default probability term structure and expected recovery rates of the underlying pool, risk free rates and implied correlations.

2. Historically observed hedging ratios. In the appendix we derive these from a regression analysis between daily tranche and underlying pool spread changes.

In the below table we provide the systemic Deltas[20] of each tranche for each calculation technique.

Exhibit 6: Model and historically estimated systemic Deltas

	Equity	junior mez	senior mez	senior	super senior	Pool
DV01	1.78	3.28	4.17	4.32	4.33	4.32
DV01 ratio	0.41	0.76	0.97	1.00	1.00	
Spread sensitivity	10.26	2.64	1.24	0.36	0.48	
Historical Systemic Delta	4.23	2.00	1.20	0.36	0.48	
Model Systemic Delta	4.39	2.65	1.14	0.61	0.92	

Source: Credit Suisse

We think it is probably best to explain Exhibit 7 by the following example. The regression in Exhibit 10 (in the Appendix) suggests the spread sensitivity of the equity tranche against changes in the underlying portfolio spread is 10.26. Furthermore, we calculate the risky PV01 of the equity tranche to be 1.78. The absolute P&L impact of a 1bp widening of the underlying portfolio spread therefore approximately leads to a P&L of

$$1.78 \times 10.26 \times \text{tranche notional}/10000.$$

Using a risky PV01 of 4.33 for the underlying portfolio, the systemic Delta, i.e. the P&L impact on the tranche relative to the P&L generated from spread changes in the portfolio is

$$1.78 \times 10.26 / 4.32 = 4.22 .$$

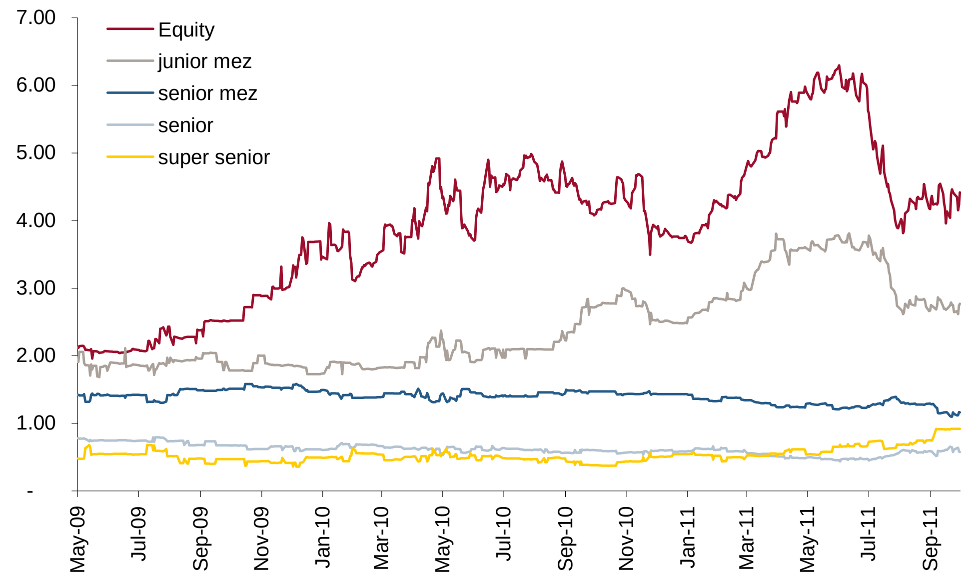
Similarly we arrive at a systemic Delta of 2.0 for the junior mez tranche. A systemic Delta neutral hedge between these two tranches should therefore have a notional in the junior mez tranche that is $4.22 / 2.0 = 2.11$ times higher than for the equity tranche.

The historically observed and the model derived ratios in some cases (e.g. equity tranche) are quite close, while for others (e.g. senior and super senior tranche) they are almost double as high under the model derived technique. We believe this could have many reasons, starting from the choice of time horizon under the regression approach (we used data going back till May 2009), over the subtle topic of autocorrelation and stationary time series to the fact that the model derived systemic Deltas are higher due to the extremely high implied correlations. The last point means, that given how wide the spreads on the senior and super senior tranches are, the model is suggesting that their respective systemic Deltas should be higher than the historically observed ones. We agree with that point as we think that the realized sensitivities in the two senior tranches should be increasing as their embedded options are getting closer to at the money (ATM).

[20] For the definition of systemic Delta, DV01 and leverage ratio see the appendix

Exhibit 7 charts the history of the model derived systemic Deltas for each tranche. We see that we have arrived at a point where the equity and junior mez tranche still have the largest systemic Deltas, however their level has decreased since August this year, when the market started to price the crisis much more from a systemic point of view resulting in increasing Deltas for the senior and super senior tranches.

Exhibit 7: Model derived systemic Deltas since May 2009



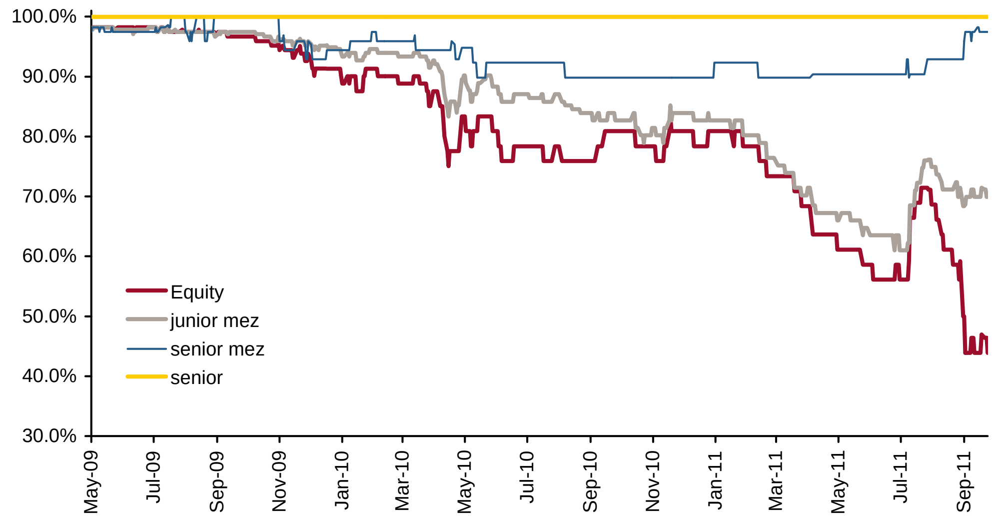
Source: Credit Suisse

Appendix

Historical Implied Base Correlations

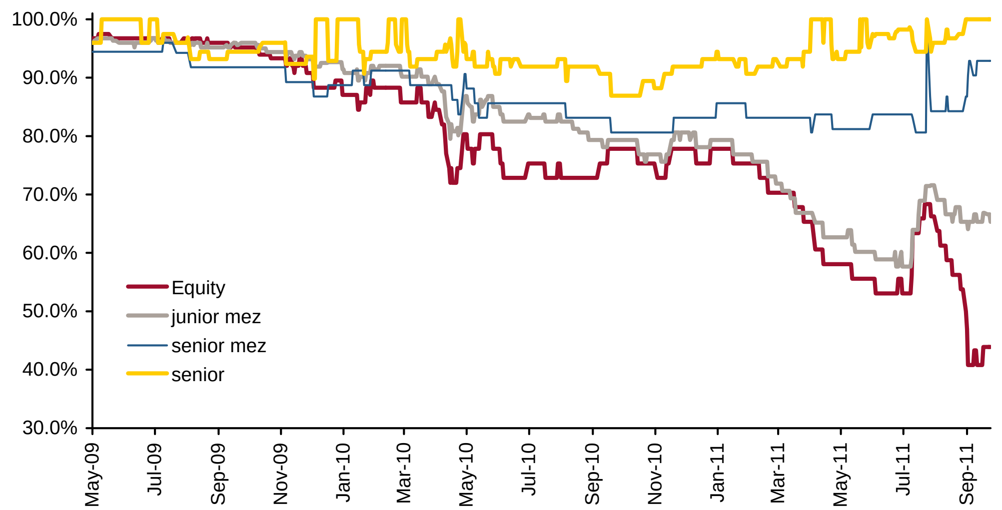
In the Gaussian copula case, a fixed recovery model is not able to produce senior tranche spreads that are wide enough to match spreads observed in the market. Models with a stochastic recovery (Exhibit 8 and 9 show this for a recovery floor at 20% and 0%) can partially resolve this, however the spreads of the super senior tranche can still not be modeled.

Exhibit 8: Time series of implied base correlations (Gaussian Copula, 20% recovery floor)



Source: Credit Suisse

Exhibit 9: Time series of implied base correlations (Gaussian Copula, 0% recovery floor)



Source: Credit Suisse

Systemic DV01 and Delta

We define the systemic Delta, systemic DV01 and leverage of a tranche according to O'Kane[21]:

$$SystemicDV01 = V_1(K1, K2) - V_0(K1, K2)$$

Where $V_0(K1, K2)$ is the value of the tranche for the current spread curve of the underlying portfolio and $V_1(K1, K2)$ is the value after bumping the spread curve of the underlying portfolio by 1bp.

$$SystemicDelta = \frac{V_1(K1, K2) - V_0(K1, K2)}{P_1} \times F$$

Where P_1 is the value of the underlying pool after bumping the spread curve by 1bp and F is the notional of the reference portfolio.

$$LeverageRatio = \frac{SystemicDelta}{F(K1, K2)}$$

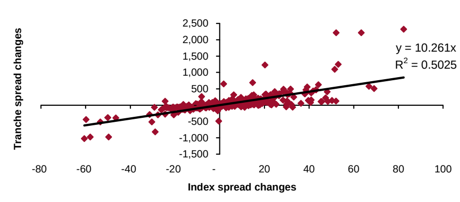
Where $F(K1, K2)$ is the tranche notional.

Historical hedging ratios

In this section we provide linear regressions between daily changes in spread of the different tranches of the euro area CDO and the underlying portfolio. This way we estimate how much a tranche spread would have reacted to a 1bp change in spread of the underlying portfolio. By multiplying these spread changes with the respective risky DV01s[22] of each tranche and the index, we can estimate the historically observed systemic DV01s and systemic Deltas, i.e. the (absolute or relative) P&L impact on each tranche given a 1bp widening in the spread of the underlying portfolio.

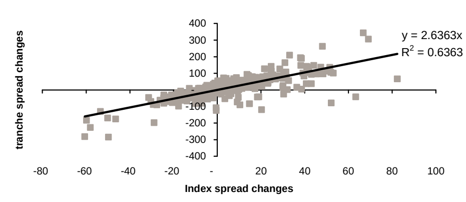
By means of cross referencing the systemic Deltas we can then calculate the historically observed hedging ratios of two respective tranches.

Exhibit 10: Equity tranche daily spread changes vs. index changes



Source: Credit Suisse

Exhibit 11: Junior Mez tranche daily spread changes vs. index changes

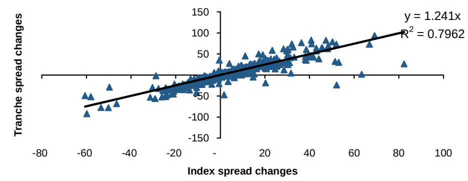


Source: Credit Suisse

[21] Modelling single-name and multi-name Credit Derivatives by Dominic O'Kane

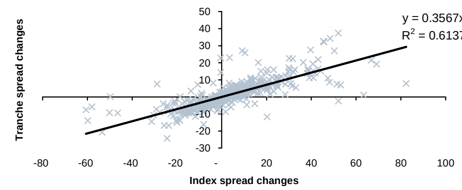
[22] The PV of a credit risky basis point running

Exhibit 12: Senior Mez tranche daily spread changes vs. index changes



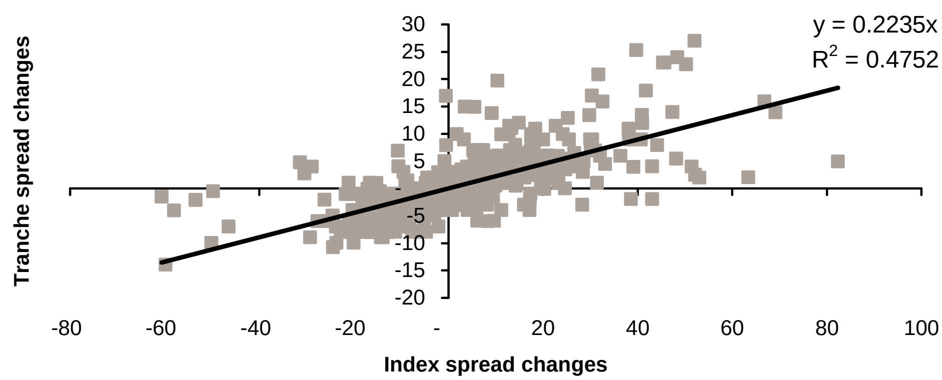
Source: Credit Suisse

Exhibit 13: Senior tranche daily spread changes vs. index changes



Source: Credit Suisse

Exhibit 14: Super senior tranche daily spread changes vs. index changes



Source: Credit Suisse

The LHP model with Gaussian Copula

The Large Homogeneous Portfolio (LHP) model under the 1-factor Gaussian Copula is based on the assumption that all credits in the underlying portfolio pool are homogeneous, i.e. have the same default probability term structure, recovery rate and default correlation with each other and that their notionals are equal. Furthermore it assumes that the portfolio is infinitely large, i.e. all idiosyncratic risk is diversified away.

These assumptions sound rather stringent, however O'Kane [23] mentions that the percentage error between the exact price and the LHP price of a tranche are typically within 5%. With the necessary caution towards any conclusions from a model we think it therefore is sufficient for our purpose of modeling the euro area CDO, building intuition about pricing of its tranches and finding relative value trade ideas.

The idea underlying this model is that the normalized asset value returns r_i of any obligor i can be described by the following linear model:

$$r_i = \sqrt{rho_i} \times Y + \sqrt{1 - rho_i} \times \varepsilon_i$$

Where rho is the default correlation between issuers, Y is the systemic factor that is N(0,1) distributed and independent of the idiosyncratic risk factors ε_i which are mutually independent and also N(0,1) distributed.

Introducing indicator functions

$$I_i = 1_{\{r_i \le C_i(T)\}}$$

[23] Modelling single-name and multi-name Credit Derivatives by Dominic O'Kane

for each issuer i, which turn 1 if r_i is smaller or equal to a threshold $C_i(T) = \Phi^{-1}(PD_i)$, where PD_i represents each issuer's individual default probability and otherwise 0, we can derive the conditional default probability of issuer i:

$$P_i(T \mid Y) = \Phi\left(\frac{C_i(T) - \sqrt{rho_i} \times Y}{\sqrt{1 - rho_i}}\right).$$

Furthermore it can be shown that under the LHP assumptions and given the conditional independence of individual issuers, due to the law of large numbers, the random variable of the portfolio loss L converges to the conditional expected loss of the portfolio. Assuming a number of n credits in the portfolio with equal notionals of 1/n, the portfolio loss L thus converges for $n \to \infty$ to

$$L \to (1 - \operatorname{Re} c) \times \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{rho} \times Y}{\sqrt{1 - rho}}\right)$$

The CDF of the portfolio loss distribution under the LHP is therefore given by:

$$P\left(L \le \frac{l}{1 - \operatorname{Re} c}\right) = \Phi\left(\frac{\sqrt{1 - rho} \times \Phi^{-1}(l) - \Phi^{-1}(PD)}{\sqrt{rho}}\right)$$

The density (PDF) can be computed by taking the derivative of the CDF with respect to l.

The LHP model with Student-t Copula

The LHP model under the Student-t Copula is very similar. The main change is that the asset returns of all obligors are multiplied by a shock variable ν / W, with W χ_ν distributed with ν degrees of freedom and independent of all variables Y and ε_i. The resultant conditional default probability under this model thus is

$$P_i(T \mid Y, W) = \Phi\left(\frac{C_i(T)\sqrt{\frac{W}{\nu}} - \sqrt{rho_i} \times Y}{\sqrt{1 - rho_i}}\right)$$

Where this time $C_i(T) = t_\nu^{-1}(PD_i)$, with $t_\nu^{-1}()$ the inverse of the CDF of a Student –t distributed random variable with ν degrees of freedom.

The CDF of the portfolio loss L (under LHP) thus becomes either

1. $$P\left(L \le \frac{l}{1 - \operatorname{Re} c}\right) = \int_0^\infty \Phi\left(\frac{\sqrt{1 - rho} \times \Phi^{-1}(l) - t_\nu^{-1}(PD) \times \sqrt{\frac{w}{\nu}}}{\sqrt{rho}}\right) f(w)\,dw$$
2. $$P\left(L \le \frac{l}{1 - \operatorname{Re} c}\right) = \Phi\left(\frac{\theta}{\sqrt{rho}}\right) + \int_{-\infty}^{-\frac{\theta}{\sqrt{rho}}} \Gamma\left(\frac{\nu}{2}, \frac{\nu \times (\theta + \sqrt{rho} \times u)^2}{2 \times C(T)^2}\right) \phi(u)\,du$$ [24]

Where f(w) is the density of W and $\theta = \sqrt{1 - rho} \times \Phi^{-1}(l)$.

[24] This second formula is derived in A note on large homogeneous portfolio approximation with Student-t copula by Lutz Schloegl and Dominic O'Kane

Tail dependency

As discussed in one of the previous sections, the Gaussian Copula has zero tail dependence for $\rho < 1$. That and the following property for the tail dependence under the Student-t copula are shown in Demarta and McNeil [25] and even extended to Copulae of generic elliptical distributions by Hult and Lindskog [26]. Due to the symmetry of the multivariate t distribution this holds both for the upper and lower tail dependence:

$$\lambda = 2 \times t_{\nu+1}\left(\frac{-\sqrt{\nu+1} \times \sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$$

The tail dependence of a Student-t Copula therefore increases with decreasing ν and increasing ρ.

Tranche valuation

Protection leg

For attachment and detachment points K1 and K2, time to maturity T, M payment dates t_i of the premium leg and a tranche notional N the PV of the protection leg is

$$V_{prot} = E[\int_0^T e^{-\int_0^t r(s)ds} N dL_t^{(K1,K2)}]$$ with $L_t^{(K1,K2)}$ the cumulative portfolio loss until t.

Assuming a deterministic risk free rate r(u) we approximate the integral by:

$$V_{prot} \approx \sum_{i=1}^{M} \frac{Z(t_{i-1}) + Z(t_i)}{2} \left(E[L_{t_i}^{(K1,K2)}] - E[L_{t_{i-1}}^{(K1,K2)}]\right)$$

Where $Z(t_i)$ are the risk free discount factors until t_i

Premium leg

The premium leg coupons are paid on the remaining notional of the respective tranche. We approximate this by the average of $1 - L_t^{(K1,K2)}$ at the beginning and end of the respective coupon period. Using the same notation as above, the value of the coupon leg thus is

$$V_{coupon} = s \times \sum_{i=1}^{M} E[e^{-\int_0^{t_i} r(s)ds} DC_i N \frac{2 - L_{t_{i-1}}^{(K1,K2)} - L_{t_i}^{(K1,K2)}}{2}]$$

Where s is the tranche spread and DC_i the daycount fraction.

This can be approximated by again assuming deterministic risk free rates:

$$V_{coupon} \approx s \times \sum_{i=1}^{M} Z(t_i) DC_i N \frac{2 - E[L_{t_{i-1}}^{(K1,K2)}] - E[L_{t_i}^{(K1,K2)}]}{2}$$

[25] The t Copula and Related Copulas: Stefano Demarta and Alexander J. McNeil

[26] Hult, H. & Lindskog, F. (2001). Multivariate extremes, aggregation and dependence in elliptical distributions.

Expected tranche loss

For recovery rate R and portfolio default frequency L_t until time t, the expected loss of a tranche with the above specifications is

$$E[L_t^{(K1,K2)}] = \frac{1}{K2-K1}\int_0^{1-R} \max(\min(l,K2)-K1,0)dQ(L_t(1-R)\le l)$$

Where Q is the probability measure of the random variable L_t.

Conservation of EL

Under a flat correlation structure (all K tranches have the same correlation), it is easy to show that the sum of the expected losses (in $) of all tranches is equal to the expected loss of the portfolio

$$\sum_{k=1}^{K} E[L_t^{(K1,K2)}]\times(K1-K2) = E[L_t]$$

O'Kane [27] shows that this property also holds true by construction under the base correlation framework.

Efficient numerical integration (Quadrature)

For the numerical calculation of both, the CDF under the Student-t copula model, as well as the EL of a tranche one needs to find an adequate Quadrature procedure. We originally used a plain vanilla trapezium rule, which can become quite expensive or inaccurate in the tail of the portfolio loss distribution. For the results shown in this note, we have used a Gaussian Quadrature scheme, which greatly improves computation speed without affecting the accuracy of the results. This is particularly crucial when calculating a history of several years of implied correlations, as the root search algorithm requires a large number of calls to the EL and therefore CDF functions.

Implied base correlation

The concept of base correlation is based on the fact that any tranche with attachment point K1 and detachment point K2 can be represented as a 0 to K2 tranche minus a 0 to K1 tranche. Furthermore, a different correlation can be assigned to each of these equity (or base) tranches.

It therefore is possible to calculate the implied correlation of the standard equity tranche (in this case it is the same as the compound correlation) and then imply the correlation of the 0 to K2 tranche by the implied correlation for the 0 to K1 tranche and the market spread of the K1 to K2 tranche. In the resultant formula one therefore needs to solve for ρ_{K2} such that:

$$\frac{K2}{K2-K1}[s_{(K1,K2)}DV01^{(0,K2)}(\rho_{K2})-V_{prot}^{(0,K2)}(\rho_{K2})] - \frac{K1}{K2-K1}[s_{(K1,K2)}DV01^{(0,K1)}(\rho_{K1})-V_{prot}^{(0,K1)}(\rho_{K1})]=0$$

Where $s_{(K1,K2)}$ is the spread of the K1 to K2 tranche and $DV01^{(0,K)}(\rho_K)$ is the PV of the coupon leg of the same tranche for a tranche spread of 1bp as a function of ρ_K.

[27] Modelling single-name and multi-name Credit derivatives by Dominic O'Kane

Root Search

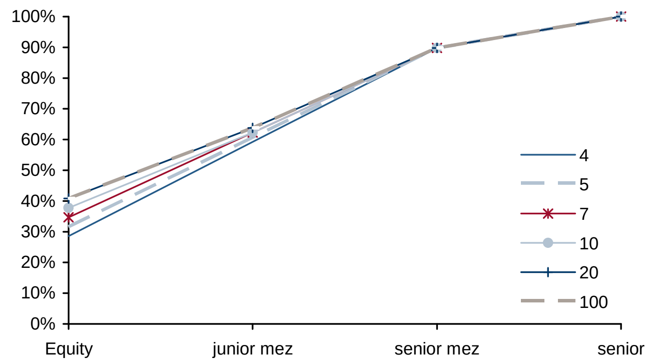
The above equation is solved by a root search algorithm. We use a simple bisection algorithm to produce the results presented in this note.

Correlation smile as a function of time and tail dependence

In the below graphs we illustrate the implied base correlations for the detachment points of each of the tranches as defined above. Exhibit 15 shows that under the Student-t copula model[28] for a wide set of the degrees of freedom parameter the result is a steeply upward sloping base correlation curve, or in other words, a steep correlation smile. Interestingly, as shown in Exhibit 16 and 17 , the correlation smile has been a lot less steep a year ago and almost flat 2 years ago. This is also summarized in Exhibit 18, which shows the correlation smiles, for the most recent data point, a year ago and 2 years ago.

Exhibit 15: Correlation smile today

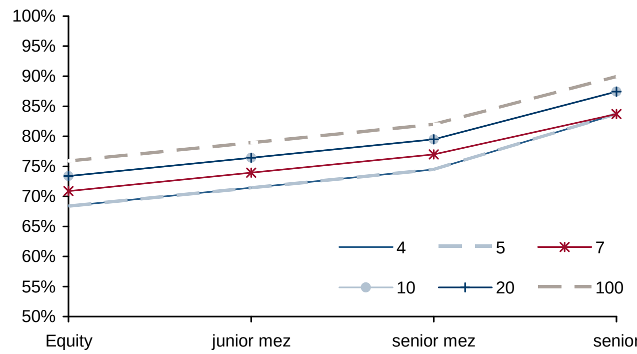
The different lines represent the implied base correlation under the Student-t copula model with each line representing a different degree of freedom



Source: Credit Suisse

Exhibit 16: Correlation smile one year ago

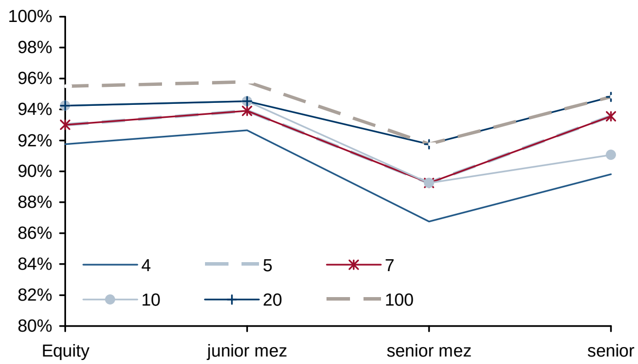
The different lines represent the implied base correlation under the Student-t copula model with each line representing a different degree of freedom



Source: Credit Suisse

Exhibit 17: Correlation smile two years ago

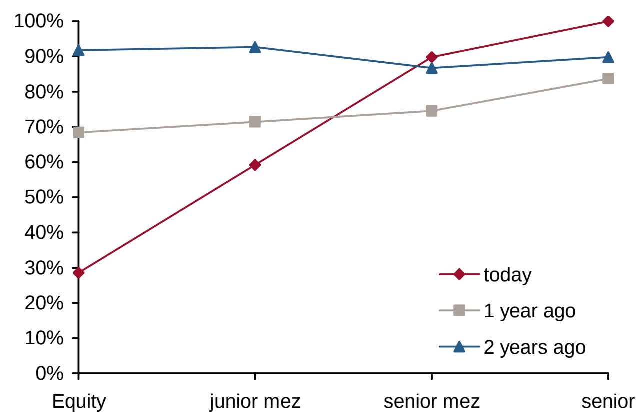
The different lines represent the implied base correlation under the Student-t copula model with each line representing a different degree of freedom



Source: Credit Suisse

Exhibit 18: Correlation smile for 4 degrees of freedom

The different lines represent the implied base correlation under the Student-t copula model with 4 degrees of freedom. Each line represents a different point in time.



Source: Credit Suisse

28 Since the Student-t Copula converges towards the Gaussian Copula model if the degrees of freedom go to infinity. We consider the case for 100 degrees of freedom therefore as a good approximation of the Gaussian model.

In the below table we additionally provide the tail dependence coefficients for a degrees of freedom parameter of 4 and the respective implied correlations displayed in Exhibit 18. It again highlights the fact that the market currently is pricing the senior tranches to have ultimate tail dependence whereas the equity tranche has very little tail dependence. This is in stark contrast to previous years in which tail dependence was mediocre and nearly flat across tranches.

Exhibit 19: Tail dependence resulting from correlation smile

Degrees of Freedom=4	Equity	junior mez	senior mez	senior
today	16%	31%	63%	99%
1 year ago	38%	40%	43%	53%
2 years ago	66%	68%	58%	63%

Source: Credit Suisse

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Disclosure Appendix

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